Math 213 – Spring 2026
Feb. 11, 2026 Exam 1

1. **(8 points)** Find the center and radius of the sphere given by:
   $x^2 + y^2 + z^2 - 4x + 6y - 2z - 11 = 0.$
   Solution: $(x-2)^2 + (y+3)^2 + (z-1)^2 = 11 + 4 + 9 + 1 = 25.$
   So the center is (2,-3,1) and the radius is 5.

2. **(21=3,4,5,5,4 points)** Let **u** = <2, -1, 3>, and **v** = <1, 4, -2>.
   a. Compute **u**·**v**.
   b. Find the cosine of the angle between **u** and **v.**
   c. Compute $\mathbf{u} \times \mathbf{v}$.
   d. Find the projection of **u** onto **v**.
   e. Find the area of the parallelogram whose sides are made up of **u** and **v.**
   Solution:
   a. **u·v**=<2, -1, 3>· <1, 4, -2>=2-4-6=-8
   b. $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|} = \dfrac{-8}{\sqrt{4+1+9}\sqrt{1+16+4}} = \dfrac{-8}{\sqrt{14}\sqrt{21}}$
   c. $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 3 \\ 1 & 4 & -2 \end{vmatrix} = -10\mathbf{i} + 7\mathbf{j} + 9\mathbf{k}$
   d. $\mathrm{proj}_{\mathbf{v}}\mathbf{u} = \dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} = \dfrac{-8}{21} < 1, 4, -2 >$
   e. **Area** = $|\mathbf{u}\times\mathbf{v}| = \sqrt{100+49+81} = \sqrt{230}$

3. Consider the line given by: **r**(*t*) = <1,2,3> + *t*<2,-1,1> and the plane given by
   $3x - y + 2z = 10$
   a. **(6 pts)** Determine whether the line intersects the plane and if so, find that intersection.
   b. **(9 points)** Find the distance from the point P(4,-1,2) to the plane.

   Solution:

   a. $3(1+2t) - (2-t) + 2(3+t)\,z = 10 \Rightarrow 7 + 9t = 10 \Rightarrow 9t = 3 \Rightarrow t = \dfrac{1}{3}$
   Then the intersection is $x = 1 + \dfrac{2}{3}; y = 2 - \dfrac{1}{3}; z = 3 + \dfrac{1}{3}$ or $\left(\dfrac{5}{3}, \dfrac{5}{3}, \dfrac{10}{3}\right)$

   b. A point in the plane is, say, **Q= (0, 0, 5**) so **PQ is <-4,1,3>**. Projecting PQ onto the normal to the plane gives us the distance from the point to the plane.

   $$|\mathrm{proj}_{\mathbf{N}}\mathbf{PQ}| = \left|\frac{\mathbf{PQ}\cdot\mathbf{N}}{|\mathbf{N}|^2}\mathbf{N}\right| = \left|\frac{<-4,1,3>\cdot<3,-1,2>}{3^2 + -1^2 + 2^2}\right|\sqrt{3^2 + -1^2 + 2^2} = \frac{7}{\sqrt{14}} = \sqrt{\frac{7}{2}}$$

4. **(12 points))** Find the distance between the skew lines
L1: **r**(*t*) = <1,0,2> + *t*<1,1,-1> and **L2**: **r**(*s*) = <2,1,0> + *s*<-1,2,2>.

Solution: Distance between skew line: find the normal to both lines, find a point on each line, then project the vector connecting the two points onto the normal.

$$\mathrm{N}=\begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ 1 & 1 & -1 \\ -1 & 2 & 2 \end{vmatrix}=4\mathrm{i}-\mathrm{j}+3\mathrm{k}$$ **Letting** ***r*****=*****s*****=** ***0*** **gives points <1,0,2> and <2,1,0>**

which are connected be the vector PQ=<1,1,-2>

Then

$$\left|\mathrm{proj}_{\mathrm{N}}\mathrm{PQ}\right|=\left|\frac{\mathrm{PQ}\cdot\mathrm{N}}{|\mathrm{N}|^2}\mathrm{N}\right|=\left|\frac{<1,1,-2>\cdot<4,-1,3>}{4^2+-1^2+3^2}\right|\sqrt{4^2+-1^2+3^2}=\frac{3}{\sqrt{26}}$$

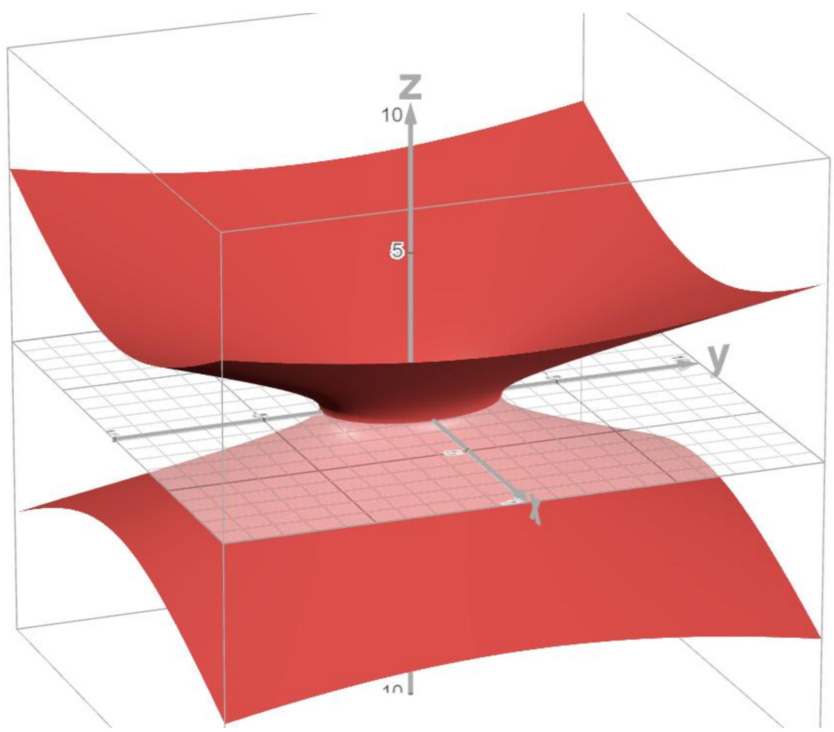
5. **(8 points)** Find the cosine of the angle between the planes
$x+y-z=10$ and $-x+2y+2z=7$

Solution: The angle between two planes = the angle between the normals so

$$\cos\theta=\frac{<1,1,-1>\cdot<-1,2,2>}{|<1,1,-1>||<-1,2,2>|}=\frac{-1}{3\sqrt{3}}$$

6. **(8 points)** Classify and sketch the surface $\dfrac{x^2}{4}+\dfrac{y^2}{9}-z^2=1$.

Solution:



From desmos: The curve is a hyperboloid.

Note that if z=0 we have an ellipse out to (2,0,0) in x and out to(0,3,0) in y .

Vertical cross sections are hyperbolas.

There is an ellipse for each value of z, so the curve is a hyperboloid of one sheet.

7. **(16=6,5,6 points)** Consider the curve: $\mathbf{r}(t) = <t^2, \sin t, e^t>$

a. Compute $\mathbf{r}'(t)$
b. Find the equation of the tangent line to the curve at $t = 0$.

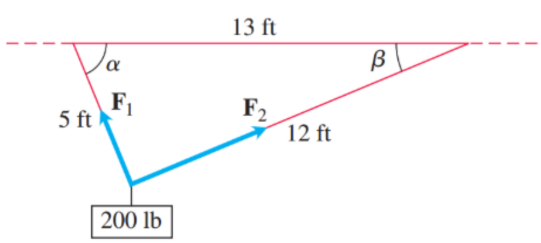
c. Compute $\int_0^1 \mathbf{r}(t)\, dt$.

Solution: a. $\mathbf{r}'(t) = <2t, \cos t, e^t>$

b. $\mathbf{r}(0) = <0, 0, 1>$ **and** $\mathbf{r}'(0) = <0, 1, 1>$ **so the tangent to the curve at** t **=0 is** $x=0;\ y=t;\ z=1+t$ or $<0,0,1>+t<0,1,1>$

c. $\int_0^1 \mathbf{r}(t)\, dt = <\frac{t^3}{3}, -\cos t, e^t>|_0^1 = <\frac{1}{3}, 1-\cos 1, e-1>$

8. **(12 points)** Consider the 200 lb weight suspended by the 2 wires as shown in the figure below. First find cos α, sin α, cos β and sin β. (Note the triangle is a right triangle). Write the relevant equations to find and then solve for $|F_1|$ and $|F_2|$.



Solution: $\cos\alpha = \frac{5}{13}; \sin\alpha = \frac{12}{13}; \cos\beta = \frac{12}{13}; \sin\beta = \frac{5}{13}$

Horizontally: $|F_1|\cos\alpha = |F_2|\cos\beta \Rightarrow 5|F_1| = 12|F_2|$

Vertically: $|F_1|\sin\alpha + |F_2|\sin\beta = 200 \Rightarrow 12|F_1| + 5|F_2| = 2600$

Thus, $60|F_1| = 144|F_2|$ **and** $60|F_1| + 25|F_2| = 13000$ **so**
$169\,|F_2| = 13000 \Rightarrow |F_2| = \frac{1000}{13}$ lbs and $|F_1| = \frac{12}{5}|F_2| = \frac{12000}{5\cdot 13} = \frac{2400}{13}$ lbs